

ria percurrit lineolam IK , corpus aliud in circulo in eadem distantia motum percurrent arcum $\frac{b}{a} \times KN$; & Area sectoris

Circuli & Trajectoriæ simul descriptæ erunt $\frac{b}{a} \times KN \times \frac{1}{2} CN$

& $KN \times \frac{1}{2} CN$, quæ duæ Areæ sunt in ratione data, scil. ut b ad a . Adeoque ubi est $a = b$, uti fit in Spirali Hyperbolica, Area sic descripta erit semper æqualis Areæ sectoris circularis in æquali tempore descriptæ.

November 24. 1713

III. *Rules for correcting the usual Methods of computing Amounts and present Values, by Compound as well as Simple Interest; and of stating Interest Accounts. Offer'd to Consideration, by Thomas Watkins, Gent. F. R. S.*

I. Of Compound Interest.

THE Supposition whereon the Method of computing by Compound Interest is founded; *viz.* That all Interest Money, Rents, &c. are or may be constantly receiv'd, and put out again at Interest, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) must needs be erroneous. Thus for Instance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the same reason, the present value of a Sum of Money payable in any time to come, will be too little; also the present value of an Annuity (being only the Amount of the difference between the Annuity, and Interest of the said present value) will be too much. But in long terms of Years, as that difference becomes less so does the Error, as the term is great.

greater; and in Fee Simples it vanishes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a just reduction of the Rate, and Annuity; (which is done by deducting so much *per Cent.* thereout, as the whole Trouble, and Charge of Management is suppos'd to amount to, and reducing the Remainder, by a Discount equivalent to the suppos'd loss of time) and then by working with the Rate so reduc'd, for Sums of Money, and with the Rate and Annuity reduc'd in the like proportion for Annuitys, according to the common Method of Compound Interest; as follows. Put r for the rate of Interest of 1 $l.$ for the Charge and Trouble of the Management of 1 $l.$ Then is $r - cr =$ the Rate after deducting the said Charge, $=$ (putting d for $1 - c$) dr . And for the Discount put t for the time lost, that is for such part of the Period of time in which the Payments are made (whether Yearly, $\frac{1}{2}$ Yearly, Quarterly, or otherwise) as is suppos'd to be spent in receiving and putting them out again at Interest. Then, dtr , being $=$ the Interest of 1 $l.$ for that time;

say, as $1 + dtr : 1 :: dr : \frac{dr}{1 + dtr} =$ (putting e for $1 + dtr$)

$\frac{dr}{e}$, which is equal to the reduc'd Rate, near enough for

practice, for which put r . But if the utmost accuracy be requir'd, the Discount itself must be made with regard to the like loss of time, which is done by a Series of Discounts rais'd

thus; $e (= 1 + tdr) : tdr :: dr : \frac{td^2r^2}{e} :: \frac{t^2d^3r^3}{e^2} : \frac{t^3d^4r^4}{e^3} \&c.$

Whence $dr \rightarrow \frac{td^2r^2}{e} + \frac{t^2d^3r^3}{e^2} + \frac{t^3d^4r^4}{e^3} \&c. =$ (putting q for

$\frac{tdr}{e}$) $1 - q + q^2 - q^3 \&c. \times dr. = \frac{dr}{e} (= \frac{dr}{1 - q \times dr}) + \frac{q^2 - q^3 \&c.}{e}$

$\times dr$, is $= r$, $=$ the true Rate reduced. Put $s = 1 + r$, $n =$ the time, $p =$ the present Sum or Value, $m =$ the Amount,

Then

Then will $1 + \frac{dr}{e} + q^2 - q^3 \&c. \times dr \Big|^n \times p = 1 + r \Big|^n \times p = p s^n$,

be exactly $= m$: But $1 + \frac{dr}{e} \Big|^n \times p = m$ is sufficient for practice.

And for the Amounts and present values of Annuitys.

Put $A =$ Annuity *per annum*.

$a =$ Annuity $\frac{1}{2}$ yearly, quarterly, &c.

$R =$ Yearly rate of Interest of 1 l .

$r =$ Rate $\frac{1}{2}$ yearly, quarterly, &c.

$t =$ Reduc'd rate yearly, $\frac{1}{2}$ yearly, quarterly, &c.

$n =$ Number of Years, $\frac{1}{2}$ years, quarters, &c.

Then will $A \frac{t}{R} = a \frac{r}{r}$ be $=$ reduc'd Annuity taken yearly

$\frac{1}{2}$ yearly, quarterly, or otherwise; and by Compound Interest

it will be $\frac{1+t \Big|^n - 1}{t} \times a \frac{t}{r} = \frac{1+t \Big|^n - 1}{r} \quad a = \frac{1+t \Big|^n - 1}{R} \quad A =$

$\frac{s^n - 1}{R} A = m$, and $\frac{s^n - 1}{R s^n} A (= \frac{m}{s^n}) = p$. Whence the

Theorems for solving all the other Cases are easily deduc'd.

And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious: when for the Amount or Value of

an Annuity, since $1+r \Big|^n = \frac{mr+a}{a} = \frac{a}{a-pr}$ are the Equations

whence Theorems for the Rate are usually deriv'd, which

by this Correction become $1+t \Big|^n = \frac{mr+a}{a} = \frac{a}{a-pr}$. That

the same t may be had on both sides the Equation, put ut

for r , and 'twill be $1+t \Big|^n = \frac{mut+a}{a} = \frac{a}{a-pu}$: then, by

the Rate assum'd as near the truth as may be, find the value

of $u (= \frac{1}{d} + tr = \frac{1}{d - td})$ and in any Theorem for the

Rate,

Rate, putting m for m , and p for p , the result will be the Rate reduced nearly : and by repeated Operations correcting r and thereby u , the true r , and thence R the whole Rate, may be found.

The only difficulty that remains, is the right assuming the Quantities c and t , the impossibility of doing which with perfect Exactness, I suppose to be the reason why neither this, nor any Method of Correction to the like purpose, has yet been taken notice of by the Writers on this Subject ; and what may therefore be very likely to be objected to this. But the same Objection I take to be of equal force against the Estimates of any other Uncertainties whatsoever, as Estates for Lives, Insurances &c.

First then, for the Quantity c , which is put for the Trouble and Charge of Management, *viz.* of collecting and placing out the Money on good Security, together with all Contingencies attending the same, as travelling Charges, Expences, Attorney's Bills, &c. of all which, the principal Article is the Charge of Collection or Receiver's Fees, which is commonly a fix'd Rate, customarily allow'd in the Place, or upon the Estate it self out of which the Purchase is made, if it be a Rent ; and for Interest Money, or any other Annuity, the like Estimate is to be made, whether the Proprietor acts for himself, or by another. Then for the Charge of placing out the Money at Interest when receiv'd ; though this be for the most part defray'd by the Borrower, yet because it highly concerns the Lender to see it be securely done, there are usual Allowances made to Agents and Scriveners, to encourage their Care and Fidelity therein ; besides the Time, Expence and Trouble of the Proprietor himself, in finishing Contracts, inspecting Securities &c. and whatever is sav'd in this Article, we must suppose to be fully made up by an Equivalent Degree of Risque in the Security.

In the next place, for the loss of time ; though 'tis also impossible for this to be exactly ascertain'd, nor perhaps so nearly as the former, since it depends very much on the Diligence of the Manager : Yet if the usual times of Payment
of

of the particular Rent or Annuity to be purchased, with a moderate degree of diligence in the Manager, and the usual indulgence practis'd by Men of Business in this Case towards one another be observ'd, a reasonable Estimate may be also made of the loss of time; In which 'tis to be noted, that Interest Money being usually paid in small Sums, when any Sum of Money to be made up of several such Payments, is intended to be put out at Interest, the whole must lie dead till the last Payment be made; also that the Principal lies dead sometimes as well as the Interest; and that on the other hand, to save time, Borrowers may be found out, and treated with during the time of Collection; but this Advantage is in a great Measure lost by the difficulty of fixing the time or *Quantum* of a Loan, till the whole be paid in. Note also, that if the Charge of Collection, or Loss of time, on the Rent, or Annuity, of any particular Estate or Place, be found to differ from that of the Interest of the Purchase Money, and so much exactness be required, as that the Computation be made with regard to such difference: It must be done, either by assuming a proper Medium for both, or more accurately thus: For the reduc'd

Rate of the $\left\{ \begin{array}{l} \text{Annuity} \\ \text{Interest-Money} \end{array} \right\}$ put $\left\{ \begin{array}{l} 2 \\ r \end{array} \right\}$ Then $r : 2 :: a : a \frac{2}{r}$,

and $r : \frac{2}{1 + r} - 1 :: a \frac{2}{r} : \frac{1 + r^n - 1}{r} a \times \frac{2}{r} = m =$

$\frac{1 + r^n}{1 + r} \times p$.

To give a Specimen of this Method in Numbers, first the Quantities c and t are to be assum'd, which are not here to be accommodated to any particular Place or Estate, but to be taken in general: And first for c the Charge of Management; the usual Rates of Collectors Fees in these Kingdoms, as I am inform'd, are *6d.* *12d.* and *18d. per Pound*, but the most usual *12d.* which is *5 per Cent.* However to be within compass, I shall take *4 per Cent.* for the Medium, including what further

R

trou-

trouble and charge may attend the Receipt of the Mony, besides Receivers Fees; and 2 *per Cent.* for all the other Charges before mention'd, in placing it out at Interest, both which make 6 *per Cent.* so that c is $=0,06$, and $d (=1-c)=0,94$. Next for t the Loss of Time; since few Annuities are paid yearly, and a Discount being given for the Loss of Time, we are to lose no more than is discounted for; therefore I choose Half-yearly Payments for the Examples, being the most usual, which with little Alteration may serve for quarterly; and considering the before-mention'd Circumstances relating to the Time, I look upon two Months the least, and seven or eight Months the most, that can well be suppos'd to be spent, one Time with another, in receiving and putting out the Mony, upon a moderate Management; between which the Medium is about four Months and half, which being $\frac{3}{4}$ of $\frac{1}{2}$ a Year, gives

$\frac{3}{4} = t$: and if $\frac{dr}{e}$ be $= r$, the yearly Rates of 4, 5, 6, 8 and

10 *per Cent.* will produce for Half-yearly Rates reduc'd of 1 *l.* 0.018539, 0.023093, 0.027616, 0.036569 and 0.0454 each =

r . But if r be $= \frac{dr}{e} + q^t - q^t \&c. \times dr$, it will be, 0.01854219,

0.023099155, 0.027627775, 0.036596289 and 0.04545235, each = r , (so that each Rate loses by this Estimate about $\frac{1}{12}$ Part.)

Whence the following Amounts, and present values of 1 *l. per Annum* computed Half-yearly, are produc'd, and compared with those of the usual Method computed yearly, to agree with the common Tables.

Tears

| Years. | Amounts of 1 l. per An. at 5 per Cent. by | | | Differences. | Amounts of 1 l. per An. at 6 per Cent. by | | | Differences. |
|--------------------------|--|---------------|------------|--------------|--|---------------|------------|--------------|
| | Comp. Int. | Co. Int. cor. | | | Comp. Int. | Co. Int. cor. | | |
| 5 | 5,52563 | 5,13104 | ,39459 | | 5,63709 | 5,22138 | ,41571 | |
| 10 | 12,57789 | 11,57846 | ,99943 | | 13,18079 | 12,07854 | 1,10225 | |
| 20 | 33,06595 | 29,85996 | 3,20599 | | 36,78559 | 32,91054 | 3,87505 | |
| 30 | 66,43885 | 58,72546 | 7,71339 | | 79,05819 | 68,83976 | 10,21843 | |
| 40 | 120,79977 | 104,30079 | 16,49898 | | 154,76197 | 130,80733 | 23,95464 | |
| 60 | 353,58372 | 289,88164 | 63,70208 | | 533,12818 | 422,01429 | 111,11389 | |
| 80 | 971,22882 | 752,53436 | 218,69446 | | 1746,5999 | 1288,2484 | 458,3515 | |
| 100 | 2610,02516 | 1905,92671 | 704,09845 | | 5638,3680 | 3864,9753 | 1773,3927 | |
| The same at 8 per Cent. | | | | | | | | |
| 5 | 5,86660 | 5,40633 | ,46027 | | 6,10510 | 5,59705 | ,50805 | |
| 10 | 14,48656 | 13,15092 | 1,33564 | | 15,93743 | 14,32680 | 1,61063 | |
| 20 | 45,76196 | 40,13759 | 5,62437 | | 57,27500 | 49,17930 | 8,09570 | |
| 30 | 113,28321 | 95,51623 | 17,76698 | | 164,49402 | 133,96428 | 30,52974 | |
| 40 | 259,05652 | 209,15737 | 49,89915 | | 442,59257 | 340,21900 | 102,37357 | |
| 60 | 1253,2133 | 920,90040 | 332,3129 | | 3034,81648 | 2062,57167 | 972,24481 | |
| 80 | 5886,9354 | 3918,0578 | 1968,8776 | | 20474,0027 | 12255,3348 | 8218,6679 | |
| 100 | 27484,5157 | 16539,0989 | 10945,4168 | | 137796,127 | 72575,3926 | 65220,7344 | |
| The same at 10 per Cent. | | | | | | | | |

| Years. | Pref. Values of 1 l. per An. at 5 per Cent. by | | | Differences | Pref. Values of 1 l. per An. at 6 per Cent. by | | | Differences |
|--------|---|---------------|--------|-------------|---|---------------|--------|-------------|
| | Comp. Int. | Co. Int. cor. | | | Comp. Int. | Co. Int. cor. | | |
| 5 | 4,32948 | 4,08343 | ,24605 | | 4,21236 | 3,97582 | ,23654 | |
| 10 | 7,72174 | 7,33314 | ,38860 | | 7,36009 | 7,00322 | ,35687 | |
| 15 | 10,37966 | 9,91935 | ,46031 | | 9,71225 | 9,30843 | ,40382 | |
| 20 | 12,46221 | 11,97753 | ,48468 | | 11,46992 | 11,06373 | ,40619 | |
| 30 | 15,37245 | 14,91902 | ,45343 | | 13,76483 | 13,41805 | ,34678 | |
| 40 | 17,15909 | 16,78200 | ,37709 | | 15,04630 | 14,78309 | ,26321 | |
| 50 | 18,25593 | 17,96190 | ,29403 | | 15,76186 | 15,57456 | ,18730 | |
| 70 | 19,34268 | 19,18247 | ,16021 | | 16,38454 | 16,29953 | ,08501 | |
| 100 | 19,84791 | 19,79231 | ,05560 | | 16,61755 | 16,59510 | ,02245 | |
| E.S. | 20,00000 | 20,00000 | ,00000 | | 16,66666 | 16,66666 | ,00000 | |

Note, That this Correction is also applicable to the Valuations of Estates for Lives, in which the first Step being to find an Equivalent in Years of Continuance, brings them to the Case of Estates for Years.

II. Of Simple-Interest.

The absurdity of the Supposition, on which the usual Method of computing present Values by Simple-Interest, is founded, *viz.* That the Rent or Annuity is constantly received, and put out again at Interest, as it becomes due; but that the Interest of the Purchase Money lies dead during the whole Term, is so apparent, and the Errors arising from it so gross, that the Writers who have laid down this Method, have at the same time caution'd against the Use of it for any more than 6 or 7 Years, the Error for that time being not considerable.

The same Supposition does also occasion the miscomputation of Amounts, or rather the misapplication of them to their proper Cases. Wherefore, since the Simple-Interest of Money is of equal Value *pro rata*, and of the same regard with Rents or Annuitys, being each the Original Profits issuing alike from a principal Stock, Estate or Value, and equally improveable; This general Rule may serve for a just Correction of this Method, *viz.* That supposing in any Case an Interest ought to be, or not to be allowed to either of those Profits, the same be done in the like Case to the other. Thus, in the Case of Debts, or Amounts of Sums of Money, Rents, or Annuitys for the time past, its usual in practice to allow no Interest to either: For though the Law, to curb the exorbitant Avarice of Usurers, and for other Reasons, does more expressly disallow Interest upon Interest for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Interest against a Tenant for Rent in Arrear; except on a *nomine pænæ* (which is now become almost obsolete) so that in this Case (putting *a* and *r* for the Annuity and Rate yearly, Half-yearly, or otherwise) as $prn + p$ is the Amount of a Sum of Money, so is an the Amount of an Annuity or Rent in Arrear, and not

$\frac{n-1}{2} r + 1 \times an$ as Arithmeticians commonly make it. But in

the Computation of present Values, or Amounts for the time to come,

come, the same being made on the Expectation of a constant regular Income of the Profits, without any extraordinary Interruption, an Interest ought to be allow'd to both, especially in present Values, which are found by setting the Amounts of both against each other: so that in these Cases, putting $x =$

$n - 1$, if $\frac{1}{2}xr + 1 \times an$, be made the Amount of a Rent or Annuity; then $\frac{1}{2}xr + 1 \times prn + p$, will be the proper Amount of a Sum of Money, and not $prn + p$: and consequently

$\frac{\frac{1}{2}xr + 1 \times an}{\frac{1}{2}xr + 1 \times rn + 1} = p$ will be the present Value of a Rent or

Annuity, the subsequent Interest being remitted on both sides in lieu of the Loss of Time and Charge of Management; which such as are apt to depreciate long Futurities, may think the properest Method of approaching the true Value; but I rather look upon the former Method of Compound Interest corrected as more exact, as well as more general, the Interest remitted in this, being in short Terms less, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the most exact of any of the Methods, that have yet been deduc'd from Simple-Interest. The reduc'd Rate may also in some Cases be properly made use of for Amounts, but not for present Values, except for short Terms; and then, since $r : a :: \frac{1}{2}xr + 1 \times rn :$

$\frac{\frac{1}{2}xr + 1 \times an}{r}$, 'twill be $\frac{\frac{1}{2}xr + 1 \times an}{r} = m = \frac{\frac{1}{2}xr + 1}{r}$

$\times prn + p$, and $\frac{\frac{1}{2}xr + 1 \times an}{\frac{1}{2}xr + 1 \times rn + 1} \times \frac{r}{r} = p$.

Examples of this Method compar'd with those of the former, will stand as follows ; in which all is computed Half-yearly, except the last Column of Compound Interest.

Amounts of 1 l. at 5 per Cent. computed 6 several Ways.

| Years. | 1 | 2 | 3 | 4 | 5 | 6 |
|--------|--------------------|----------------------|---|--|------------------------|------------------------------|
| | <i>Simple Int.</i> | <i>Id. for Bonds</i> | <i>Sim. Int. cor.</i> | <i>Id. by the red. rat.</i> | <i>Co. Int. cor.</i> | <i>Comp Int.</i> |
| | $prn + p = m$ | $Id. till prn = p$ | $\frac{i \times r + i \times}{prn + p = m}$ | $\frac{i \times r + i \times pr}{+ p = m}$ | $i + r^n \times p = m$ | $\frac{N}{+ R} \times p = m$ |
| 5 | 1,25 | 1,25 | 1,27813 | 1,25501 | 1,25655 | 1,27628 |
| 10 | 1,5 | 1,5 | 1,61875 | 1,56338 | 1,57892 | 1,62889 |
| 20 | 2, | 2, | 2,48750 | 2,34021 | 2,49300 | 2,65330 |
| 30 | 2,5 | 2, | 3,60625 | 3,33048 | 3,93625 | 4,32194 |
| 40 | 3, | 2, | 4,97500 | 4,53419 | 6,21504 | 7,04000 |
| 60 | 4, | 2, | 8,46250 | 7,58194 | 15,49408 | 18,67919 |
| 80 | 5, | 2, | 12,95000 | 11,48345 | 38,62672 | 49,56144 |
| 100 | 6, | 2, | 18,43750 | 16,23874 | 96,29634 | 131,50126 |

Amounts of 1 l. at 6 per Cent. by the same Theorems.

| | | | | | | |
|-----|-----|-----|---------|----------|-----------|-----------|
| 5 | 1,3 | 1,3 | 1,3405 | 1,31063 | 1,31328 | 1,33822 |
| 10 | 1,6 | 1,6 | 1,7710 | 1,69758 | 1,72471 | 1,79084 |
| 20 | 2,2 | 2, | 2,9020 | 2,70048 | 2,97463 | 3,20713 |
| 30 | 2,8 | 2, | 4,3930 | 4,00870 | 5,13039 | 5,74349 |
| 40 | 3,4 | 2, | 6,2440 | 5,62223 | 8,84844 | 10,28572 |
| 60 | 4,6 | 2, | 11,0260 | 9,76525 | 26,32086 | 32,98769 |
| 80 | 5,8 | 2, | 17,2480 | 15,12954 | 78,29488 | 105,79599 |
| 100 | 7, | 2, | 24,9100 | 21,71510 | 232,89852 | 339,30208 |

Amounts of 1 l. at 10 per Cent. by the same Theorems.

| | | | | | | |
|-----|-----|-----|---------|----------|-----------|------------|
| 5 | 1,5 | 1,5 | 1,6125 | 1,54749 | 1,55970 | 1,61051 |
| 10 | 2, | 2, | 2,4750 | 2,30157 | 2,43268 | 2,59374 |
| 20 | 3, | 2, | 4,9500 | 4,42951 | 5,91793 | 6,72750 |
| 30 | 4, | 2, | 8,4250 | 7,38381 | 14,39643 | 17,44940 |
| 40 | 5, | 2, | 12,9000 | 11,16449 | 35,02190 | 45,25925 |
| 60 | 7, | 2, | 24,8500 | 21,20493 | 207,25717 | 304,48165 |
| 80 | 9, | 2, | 40,8000 | 34,55084 | 1225,5335 | 2048,4003 |
| 100 | 11, | 2, | 60,7500 | 51,20222 | 7258,5398 | 13780,6127 |

Years

| Years. | Pres. Values of 1 l. per An. at 5 per Cent. by | | | Pres. Values of 1 l. per An. at 6 per Cent. by | | |
|--------|--|---------------|------------|--|---------------|------------|
| | Sim. In. cor. | Co. Int. cor. | Comp. Int. | Sim. In. cor. | Co. Int. cor. | Comp. Int. |
| 5 | 4,35208 | 4,08343 | 4,32948 | 4,23349 | 3,97582 | 4,21236 |
| 10 | 7,64478 | 7,33314 | 7,72174 | 7,25579 | 7,00322 | 7,36009 |
| 15 | 10,10821 | 9,91935 | 10,37966 | 9,39341 | 9,30843 | 9,71225 |
| 20 | 11,95980 | 11,97753 | 12,46221 | 10,92350 | 11,06373 | 11,46992 |
| 30 | 14,45407 | 14,91902 | 15,37245 | 12,87275 | 13,41805 | 13,76483 |
| 40 | 15,97990 | 16,78200 | 17,15909 | 13,99744 | 14,78309 | 15,04630 |
| 50 | 16,96682 | 17,96190 | 18,25593 | 14,69544 | 15,57456 | 15,76186 |
| 70 | 18,10986 | 19,18247 | 19,34268 | 15,47252 | 16,29953 | 16,38454 |
| 100 | 18,91525 | 19,79231 | 19,84791 | 15,99739 | 16,59510 | 16,61755 |
| F.S. | 20,00000 | 20,00000 | 20,00000 | 16,66666 | 16,66666 | 16,66666 |

The Theorems to the preceding Columns of Amounts (of which the fourth and fifth are infinitely variable in the Result, by assuming *c* and *t* in the reduc'd Rates at Pleasure) may serve to answer all the simple Cases of Amounts that occur in Business: To instance in some,

1. The first Column contains the Amounts of such Debts, or Sums of Money as carry a simple Interest till the Principal be paid.

2. The second Column answers the common Case of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column answers the Case of a Security or joynt Obligor, that has duly and constantly paid the Interest, and at last the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to save him harmless, against whom he may make his Charge from this Column.

4. In case the Parties shall agree that the Debt shall lie for any time certain, or uncertain; and for the much greater ease, advantage and satisfaction of both of them, no Interest to be call'd for, till the Principal it self is paid; but to carry Interest as it becomes due, the Lender allowing for the time and charge he must have been at, in receiving and putting out his Interest, the fourth Column will fit this Case, or else the fifth

as a greater or less degree of Lenity is agreed upon in favour of the Borrower.

5. The fifth Column is also proper in the following Case, *viz.* if it be demanded, what Estate in reversion, after a certain number of Years, any Sum in Hand will Purchase; the first step being to find the Amount of that Sum to the time the Reversion commences, its had in this Column.

6. The last Column gives the Amount of a Sum of Money, according to the common Method of Compound Interest, but being computed with that extraordinary rigor as has been said, (except some small allowance for the loss of time, by being done Yearly) 'tis hardly suitable to any Case.

Other Cases might be enumerated, to which the foregoing Theorems might be equitably apply'd; besides such extraordinary ones, wherein it may appear to Arbitrators, or a Court of Equity, that either Partie deserves Favour, either by way of Compensation for Injuries suffer'd from the other, by means of any fraudulent or oppressive Practices, or otherwise, for which no other redress is provided.

III. Of Interest Accounts.

The Inequality of the usual Method of stating Interest Accounts, as practis'd in our Courts of Equity, will best appear by an Example, for which I shall take the common general Case of an Interest Account to be stated on a Mortgage, *viz.* suppose 1000 *l.* to be let out at 6 *per Cent.* on a Mortgage of 120 *l. per annum*, payable Half Yearly, and the Mortgagee after five Years end, to have Possession till the Arrear of Interest, accruing Interest and Principal be discharged : *Quære*, How long that will be? supposing also, for the sake of brevity in the Account, the Payments to be equally, and punctually made as they become due. By the Chancery Method, the Rent is first apply'd to discharge the Arrear of Interest; and then the remainder of every Half-Year's Rent, after deducting the same Half-Years Interest, is apply'd towards the Discharge of

of the Principal, and thereby the Principal and Interest continually lessens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is discharged, pays Comp. Interest, with the utmost rigour, for so much *per annum* of the Rent, as exceeds the Interest of the whole Principal Money, and receives but Simple Interest for his Debt; which, however strange it may seem, is easily prov'd, by applying the proper Theorems of Simple and Compound Interest to this Case, in which the Annuity, Principal Money, Rate and Arrear of Interest are given, and the time requir'd; the result being the same with that of the Chancery Method, except a very small difference only when any part of the time is express'd by a Fraction: *viz.* putting L for Logarithm, $\alpha = a - pr = 30$, $s = 1 + r = 1.03$, $t =$ time of contracting the Arrear $= 10$ Half-years, $n =$ any number of $\frac{1}{2}$ Years spent in discharging the whole or any part, $N =$ number of Years required; the Equation for the Arrear will be $prt + prn = an$; and for the Principal and accruing Interest $prn + p = prn + \frac{s^n - 1}{r} \times \alpha$. Whence

$$\frac{prt}{2\alpha} + \frac{La - L\alpha}{2Ls} = N = 16,7249 \text{ Years} = \text{the time demanded;}$$

$$\text{i.e. } \frac{prt}{2\alpha} = 5 \text{ Years} = \text{the time of discharging the Arrear, and}$$

$$\frac{La - L\alpha}{2Ls} = 11,7249 \text{ years,} = \text{the time in which the Prin-}$$

incipal and accruing Interest is discharged; during which it is evident, the Mortgagee pays full Compound Interest for 60 *l.* *per annum* of the Rent. For the Correction of which inequality, in the first Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as necessary to be always observ'd, *viz.* That Amounts of the Produce on each side be stated separately, and set against each other in the Account, in order to a Ballance. And in the common Cases of Mortgages, Government, and

Stock Securities, &c. where the Debt is paid off by a Rent, Annuity, Pension, Dividend, or other Payments issuing in the same manner, and with the like trouble, charge, &c. as Interest Money does; I presume this Rule will also be easily admitted, *viz.* That the same equitable Advantage be impartially allow'd on both sides; for which the Method of Simple Interest, as corrected under the foregoing Head, seems truly adapted; whereby the Original Profits on each side are suppos'd to be deem'd, either as Interest, or else as Principal Money; and since the Amounts both of an Annuity and Sum of Money, for the time past, as there stated, on the first of these Suppositions (t being there $= 0$) are likewise vouch'd by our Laws, and the practice of our Courts, to be good when separately us'd; I think its very evident, that the Account ought to be stated by setting those Amounts against each other thus,

$$prt + prn + p = an, \text{ (whence } \frac{1 + tr}{2\alpha} p = N, = 21,6666$$

Years) and that this Method is most proper for general Use, in the Cases mention'd: Unless it should be thought fit, in consideration of the various Ways found out for the ready improvement of Money, to allow a further Advantage on both sides, by charging the Original Profits as Principal Money, and giving a Simple Interest there to, which still falls short of the Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, *viz.* either by applying an Amount of Rent to pay off the Arrear first, and afterwards another Amount of Rent to discharge the Principal, and accruing Interest; or else by letting the Profits with all Arrears and other Charges run on at Simple Interest on each side, till the end of the Term: *viz.* putting $x = n - 1$, $y = t - 1$, $\alpha = a - pr$, $f = 2 - r \times \alpha$, $g = f - 2prt^2$, $\mu = \frac{1}{2}yr + \frac{1}{2} \times prt =$ Arrear of Interest; by the first of these 'twill be, for the Arrear, $\mu + prt^2n = \frac{1}{2}xr + \frac{1}{2} \times \alpha n$, and for the Principal and accruing Interest, $\frac{1}{2}xr + \frac{1}{2} \times \alpha n = p$; Whence
 $\sqrt{8\mu\alpha}$

$$\frac{\sqrt{8\mu ar + g^2} - \sqrt{8par + f^2} - f + g}{4ar} = N = 18,7653$$

Years. By the other 'twill be, for the whole, $\mu + ptr^2n + p =$
 $\frac{1}{2}xr + 1 \times an$, Whence $\frac{\sqrt{\mu + p \times 8ar + g^2} - g}{4ar} = N =$

18,1648 Years. The Lender will have to alledge for the first of these two Ways, that as the Rent is not hindred by any other parallel Charge from making the utmost produce it can, so for that reason ought his Principal Mony to have the Advantage of the Arrears being first discharged, which also agrees with the Chancery Method in this particular.

Lastly, another way of stating this Account, may be taken from that Notion of Simple-Interest, whereby the Annuity only is charged as Principal Mony: and then 'twill be, for the Arrear, $prt + prn = an$, and for the Principal and accruing

Interest, $\frac{1}{2}xr + 1 \times an = p$; Whence $\frac{\sqrt{8par + f^2} - g}{4ar} (=$

$$\frac{prt}{2a} + \frac{\sqrt{8par + f^2} - f}{4ar}) = N = 17,3072 \text{ Years ; which}$$

appears to be same with the Chancery Method, only that the Compound Interest in that, is turned into Simple in this; and as it still retains part of the same inequality, to the Advantage of the Borrower, it seems only fit to be observed in such Cases wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, arising by extraordinary Risque or Industry. But since such a Rule of distinction is hardly possible to be reduc'd to general practice, the use of this Theorem seems restrained to such Cases only, wherein the Parties themselves, or a Court of Equity shall think it reasonable.

For a further illustration of these Rules, the following Specimen is added, which shews at sight, how the Results of the several Methods differ, as the Rent, Arrear, or Rate of Interest, is greater or less, and consequently of how much more

or less concern it is to the Parties, as well as to the due Administration of Justice, to have regard thereto.

| The time requir'd in ————— | Years (computed Half-yearly.) | | | |
|---|-------------------------------|------------------|----------|----------|
| To discharge a Mort. of 1000 l. by a Rent of ————— | 120 l. per annum. | 90 l. per annum. | | |
| At the Rate of Int. <i>per annum</i> of ————— | 5 per C. | 6 per C. | 5 per C. | 6 per C. |
| No Arrear of Interest. | | | | |
| By the Chancery Method ————— | 10,9141 | 11,7249 | 16,4205 | 13,5335 |
| By the same turning the Comp. Int. into Sim. ————— | | | | |
| By Simp. Int. cor. } Princ. and { contin ^d at Int. | 11,2579 | 12,3072 | 17,5335 | 20,7400 |
| the Original Pro- } the Arrear { first discharg'd | | | | |
| fits charged as } Interest ————— | 14,2857 | 16,6666 | 25,0000 | 33,3333 |
| 5 Years Arrear of Interest. | | | | |
| By the Chancery Method ————— | 14,4855 | 16,7249 | 22,6705 | 28,5835 |
| By the same turning the Comp. Int. into Sim. ————— | | | | |
| By Simp. Int. cor. } Princ. and { contin ^d at Int. | 14,8293 | 17,3072 | 23,7835 | 30,7400 |
| the Original Pro- } the Arrear { first discharg'd | 15,3191 | 18,1648 | 24,7148 | 32,7064 |
| fits charged as } Interest ————— | 15,5897 | 18,7653 | 25,4899 | 34,8052 |
| 10 Years Arrear of Interest. | | | | |
| By the Chancery Method ————— | 17,8571 | 21,6666 | 31,2500 | 43,3333 |
| By the same turning the Comp. Int. into Sim. ————— | | | | |
| By Simp. Int. cor. } Princ. and { contin ^d at Int. | 18,0569 | 21,7249 | 28,9205 | 38,5835 |
| the Original Pro- } the Arrear { first discharg'd | 18,4007 | 22,3072 | 30,0335 | 40,7400 |
| fits charged as } Interest ————— | 20,3161 | 25,5998 | 33,6320 | 48,0874 |
| 15 Years Arrear of Int. | | | | |
| By the Chancery Method ————— | 21,2895 | 27,5587 | 36,1932 | 53,8107 |
| By the same turning the Comp. Int. into Sim. ————— | | | | |
| By Simp. Int. cor. } Princ. and { contin ^d at Int. | 21,4286 | 26,6666 | 37,5000 | 53,3333 |
| the Original Pro- } the Arrear { first discharg'd | 21,6283 | 26,7249 | 35,1705 | 48,5835 |
| fits charged as } Interest ————— | 21,9721 | 27,3072 | 36,2835 | 50,7400 |
| By Simp. Int. cor. } Princ. and { contin ^d at Int. | 26,1353 | 34,2913 | 43,9764 | 65,8386 |
| the Original Pro- } the Arrear { first discharg'd | 28,0209 | 37,7141 | 48,5159 | 74,5664 |
| fits charged as } Interest ————— | 25,0000 | 31,6666 | 43,7500 | 63,3333 |

I must also observe for the sake of such as are unacquainted with specious Arithmetic, that though for brevity's sake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the same Accounts may be stated after the Chancery manner it self, according to the several Principles before deliver'd, and with the same Results, with this only caution, that (instead of a continual deduction of the Rent or Annuity out of the Principal and Interest of the Debt, which occasions the Error before mention'd) the preceding General Rule of stating separate Amounts be observ'd, which may be done by continually adding the profits together on each side, in the same manner, as if the Parties were

to

to make a separate Charge against each other, which is the rather to be noted, as being the only Course that can be taken, in case the Sums or Times of payment should differ, but the respective Results will notwithstanding be analogous to the above Examples.

All which is submitted to the consideration of more discerning Judgments, especially the Applications of the Rules to particular Cases, for exemplifying the Theorems. But if any of those Rules or Theorems should be objected against, merely because they tend to introduce some Alterations in the present practice; I shall for answer only add, with submission, to what is before said, that in former Ages, when our Laws relating to these matters had their rise, (the Profits of *England* arising chiefly from Husbandry and Tillage, and little from Trade,) the Cash of the Kingdom was but low, the Rates of Interest very high to the Advantage of Usurers, and those ways for the ready Improvement of Money accomodated to all Peoples use, not known; (much like to which we are told was the State of the Jewish Affairs, when they were forbidden to take Usury of any but Strangers.) But latter Ages have produc'd vast Alterations in all these Respects, which having happen'd by insensible degrees, may be one reason why neither our Legislature, nor Courts of Judicature have yet taken such notice thereof, as time and leisure, with the Tender of proper and practical Methods of Computation, may hereafter induce them to do.

F I N I S.

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